4.NF Doubling Numerators and Denominators

Alignments to Content Standards: 4.NF.A.2

Task

a. How does the value of a fraction change if you double its numerator? Explain your answer.

b. How does the value of a fraction change if you double its denominator? Explain your answer.

IM Commentary

The purpose of this task is to assess whether students understand the meaning of the numerator and the denominator in a fraction. This task is not appropriate for a high-stakes summative assessment, but it could be very helpful in gauging students' flexibility with the meaning of fractions. This task would benefit from representative examples of student work. This task would work well as the culmination of a set of tasks where students explore the meaning of fractions.

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Solution

a. Doubling the numerator doubles the value of the fraction. Some examples of grade-appropriate explanations (in adult language):
  • The numerator tells you how many pieces there are. If you double the numerator,

you double the number of pieces, so you will have twice as many pieces. For example, if you have $\frac{5}{8}$, which is 5 pieces, and double the numerator, that gives you

$$2 \times \frac{5}{8} = \frac{2 \times 5}{8} = \frac{10}{8}$$

which is 10 pieces. Since the denominator stays the same, the size of the pieces stays the same. Since you have twice as many of the same size pieces, the value of the fraction is doubled when you double the numerator.

- If you double the numerator in a unit fraction, that means you have two unit fractions. For example, doubling $\frac{1}{3}$ means taking $\frac{1}{3}$ two times which is $\frac{2}{3}$. If you double the numerator of any fraction, you double the number of unit fractions. For example, if you have $\frac{4}{3}$ that is 4 thirds, and doubling the numerator gives you $\frac{8}{3}$ which is 8 thirds. So the value of the fraction is doubled when you double the numerator.

b. Doubling the denominator cuts the value of the fraction in half. Some examples of grade-appropriate explanations (in adult language):

- The denominator tells you how many equal-sized pieces make a whole. If you double the denominator, you double the number of pieces needed to make a whole, so each piece will be half as big. For example, suppose you have $\frac{5}{3}$ and the denominator doubles so you have $\frac{5}{6}$. Sixths are half as big as thirds:

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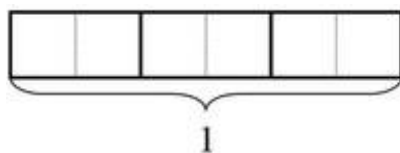
Since the numerator stays the same, the number of pieces stays the same, but each piece is half as big. So the value of the fraction is half what it was when you double the denominator.

- If you double the denominator in a unit fraction, that means your unit fraction is half as big. For example, doubling the denominator in $\frac{1}{3}$ gives $\frac{1}{6}$, and if it takes 6 equal-sized pieces to make a whole, then they must be half as big as if it only takes 3 equal-sized pieces to make a whole. If you double the denominator of any fraction, the number of unit fractions stays the same but each unit fraction is half as big. So the value of the fraction is half what it was when you double the denominator.