Illustrative Mathematics

5.NBT Kipton's Scale

Alignments to Content Standards: 5.NBT.A.1

Task

a. Kipton has a digital scale. He puts a marshmallow on the scale and it reads 7.2 grams. How much would you expect 10 marshmallows to weigh? Why?

b. Kipton takes the marshmallows off the scale. He then puts on 10 jellybeans and then scale reads 12.0 grams. How much would you expect 1 jellybean to weigh? Why?

c. Kipton then takes off the jellybeans and puts on 10 brand-new pink erasers. The scale reads 312.4 grams. How much would you expect 1,000 pink erasers to weigh? Why?

IM Commentary

The purpose of this task is to help students

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left (5.NBT.1)

By setting the task in the context of weighing objects and bundles of 10, 100, and 1,000 objects, it helps students visualize that bundling 10 units of a given place value will create 1 unit of the next highest place value. For example, taking 10 of an item that weighs 4.2 grams will result in 42 grams because it is 10 groups of 4 grams that weights 40 grams all together, and 10 groups of 0.2 grams that weigh 2 grams all together. The task allows students to explore both the structure of our place value system and how we use that to efficiently multiply and divide powers of 10.

Though this task is written as questions to be discussed or answered by students, the parts of this task are most valuable as a set of scenarios to be infused throughout a unit on place value. The teacher can actually bring in a digital scale and go through a series of explorations with students. It might be natural to start with students weighing one object and making predictions about how much 10 or 100 of these objects would weigh. From there, the teacher may want to have students start weighing sets of 10 or 100 objects and work backwards to think about the weight of 1 object.

The digital scale has several advantages: the weight will always appear in decimal form, thereby making it perfect for students to start reasoning about shifts in decimal place value from a more intuitive place. Moreover, the digital scale will not always show the expected answer. For example, something that weighs 3.5 grams alone might weigh 35.4 grams (rather than 35 grams) when taken as a group of 10. This slight difference provides an excellent opportunity to talk about rounding error. It also requires students to think beyond the rules of sliding a decimal point to the right when multiplying by powers of 10.

If students were still developing the rules for multiplying and dividing by powers of 10, this task would also incorporate MP8, Look for and express regularity in repeated reasoning. It also incorporates parts of MP6, Attend to precision, in that a higher-level discussion will push students to reason why one marshmallow might weigh 7.2 grams, but 10 marshmallows might weigh 7.19 grams. See the solution for part (a) for further elaboration.

Edit this solution

Solution

Solution:

a. 10 marshmallows should weigh 72 grams. Students might use repeated addition, multiplication or reason that each digit's place value will be multiplied by a factor of 10:

$$\begin{aligned}10 \times (7 + 0.2) &= \\ (10 \times 7) + (10 \times 0.2) &= \\ 70 + 2 &= \\ 72\end{aligned}$$

If students have had practice using digital scales in class, some students might respond with guesses that are close such as 73 grams or 72.4 grams because the original 7.2 grams may have been rounded to the nearest tenth. If, for example, the original actual weight were 7.24 grams, then the weight of ten marshmallows on a scale that rounds to the nearest tenth of a gram would be 72.4 grams.

b. 1 jellybean should weigh 1.2 grams. Students may come to the solution a number of ways.

- Students might notice that dividing 12 by 10 could be expressed in fraction form $\frac{12}{10}$. Students might further reason that

$$\frac{12}{10} = \frac{10}{10} + \frac{2}{10}$$

or 1.2.

- Students might alternately reason that $12 \div 10$ is the same as

$$(10 \div 10) + (2 \div 10) = 1 + 0.2 = 1.2$$

- Students might also use the rule that they are beginning to develop about sliding the decimal point one place to the left to divide by 10.

c. 1,000 pink erasers should weigh about 31,240 grams. Students may come to the solution a number of ways:

- A student might reason that first, it would be necessary to find out what 100 erasers would weigh by multiplying by a factor of 10 and then further multiplying by another factor of 10 to find out what 1,000 erasers would weigh. This is represented by

$$312.4 \times 10 \times 10$$

In this solution method, the student reasoned that multiplying by a factor of 10 and then another factor of 10 would be the same as multiplying by a factor of 100:

$$\begin{aligned} 312.4 \times 10 \times 10 &= \\ 312.4 \times 100 &= \\ 100 \times (300 + 10 + 2 + 0.4) &= \\ 30,000 + 1,000 + 200 + 40 &= \\ 31,240 & \end{aligned}$$

- Another student might want to find the weight of a single eraser. If 10 erasers weigh 312.4 grams, then one eraser must weigh 31.24 grams. From there the student could multiply the weight of one eraser by a factor of 1,000:

$$\begin{aligned} 1000 \times (30 + 1 + 0.22 + 0.04) &= \\ 30,000 + 1,000 + 200 + 40 &= \\ 31,240 & \end{aligned}$$