

6.EE Seven to the What?!?

Alignments to Content Standards: 6.EE.A.1

Task

- What is the last digit of 7^{2011} ? Explain.
- What are the last *two* digits of 7^{2011} ? Explain.

IM Commentary

At first glance, this might seem like an impossible problem because the number 7^{2011} is much too large to evaluate even with a calculator. It requires a starting point, and, to many, such a starting point might not be obvious. Students who successfully answer this problem must "look for and express regularity in repeated reasoning" (MP8), and so the purpose of this task is to give students an opportunity to engage in MP8 and to practice working with positive integer exponents. There are two key steps involved in the solving this problem:

- Recognizing (or at least presuming) that the last two digits of 7^{2011} can be found without calculating the number 7^{2011} .
- Identifying a pattern in the last two digits of successive powers of 7 (which hopefully appears quickly!).

Once students see that there is a pattern to the last two digits of successive powers of 7, they can easily guess the correct answer. A complete explanation for why we know the pattern will continue will probably not be found by a 6th grader, but because it is based on place value, it could be understood by sixth graders who have a good understanding of the base-ten number system.

It would be fun to replace the exponent 2011 with the current year, which doesn't

change the substance of the problem in any way. This task is only intended for instructional purposes and would be completely inappropriate for high-stakes assessment (in case anyone wonders).

This task was adapted from problem #22 on the 2011 American Mathematics Competition (AMC) 8 Test. For the 2011 AMC 8, which was taken by 153,485 students, the multiple choice answers for the problem which asked students to find the ten's digit of 7^{2011} had the following distribution:

Choice	Answer	Percentage of Answers
(A)	0	16
(B)	1	16
(C)	3	14
(D)*	4	21
(E)	7	21
Omit	-	12

Of the 153485 students: 72,648 (47%) were in 8th grade, 50,433 (33%) were in 7th grade, and the remainder were less than 7th grade.

Solutions

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Solution: 1 Making a Table

Below is a table listing the first 8 powers of 7 along with the final digits of these powers:

Power of 7	Result	Last two digits
7^1	7	07

7^2	49	49
7^3	343	43
7^4	2401	01
7^5	16,807	07
7^6	117,649	49
7^7	823,543	43
7^8	5,764,801	01

Notice that the last two digits are following a simple repeating pattern:

07, 49, 43, 01.

We are interested in the 2011th number in this sequence. The 2000th power will go through this sequence of 4 numbers $2000 \div 4 = 500$ times. The 2008th power will go through this sequence of 4 numbers 2 more times. Thus 7^{2009} will end in 07, 7^{2010} ends in 49 and 7^{2011} ends in 43. So we have an answer to both questions:

- The ones place of 7^{2011} is a 3.
- The tens place of 7^{2011} is a 4.

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Solution: 2 Reasoning with place value

a. To find successive powers of 7, we can multiply by 7 repeatedly, with 2011 factors of 7 in all to get 7^{2011} . We are only interested in the ones place and start with 7^3 as an example:

$$\begin{aligned}
 7^3 &= 7 \times 7^2 \\
 &= 7 \times 49 \\
 &= 7 \times (4 \times 10 + 9) \\
 &= (7 \times 4 \times 10) + (7 \times 9).
 \end{aligned}$$

Since $7 \times 9 = 63$, the ones place of 7^3 will be 3. Notice that the 4 tens in 49 do not play a role in determining the ones place of the product. This makes sense as these tens will only influence the higher place values in the product. So when we are looking to find the ones digit of 7^{2011} we only need to pay attention to the ones digit in each successive power. Starting with the first power these are 7, 9, 3, 1 and then the sequence repeats. Since

$$2011 = 502 \times 4 + 3$$

this means that for 7^{2011} we will go completely through this sequence of last digits 502 times and then land on the third one which is 3. So the last digit of 7^{2011} is 3.

b. We can apply the same reasoning to find the last two digits of 7^{2011} . Taking 7^4 as an example, note that

$$\begin{aligned} 7^4 &= 7 \times 7^3 \\ &= 7 \times 343 \\ &= 7 \times (3 \times 100 + 43) \\ &= (7 \times 3 \times 100) + (7 \times 43). \end{aligned}$$

As above, the only part of this expression which contributes to the tens and ones digits is 7×43 . We have $7 \times 43 = 301$ and so the digit in the hundreds place of 7^4 is 0 and the digit in the ones place of 7^4 is 1. At this point we enter a nice pattern as we see by studying at $7^5 = 7 \times 7^4$: since we know 7^4 has a zero in the hundreds place and a 1 in the ones place, when we multiply by 7 we will get a 0 in the hundreds place and a 7 in the ones place. So we will be back to where we started with the same tens digit and ones digit as 7^1 . This pattern will continue so the first, fifth, ninth (and so on) powers of 7 end in 07. Since

$$2009 = 502 \times 4 + 1$$

this means that the last two digits of 7^{2009} are 07. For 7^{2010} we have to multiply by 7 so this gives 49 for the last two digits. For 7^{2011} we multiply by 7 again giving 43 for the last two digits.



