6.EE The Djinni's Offer

Alignments to Content Standards: 6.EE.A.1

Task

After opening an ancient bottle you find on the beach, a Djinni appears. In payment for his freedom, he gives you a choice of either 50,000 gold coins or one magical gold coin. The magic coin will turn into two gold coins on the first day. The two coins will turn into four coins total at the end of two days. By the end or the third day there will be eight gold coins total. The Djinni explains that the magic coins will continue this pattern of doubling each day for one moon cycle, 28 days. Which prize do you choose?

When you have made your choice, answer these questions:

- The number of coins on the third day will be $2 \times 2 \times 2$. Can you write another expression using exponents for the number of coins there will be on the third day?
- Write an expression for the number of coins there will be on the 28th day. Is this more or less than a million coins?

IM Commentary

The purpose of this task is to introduce the idea of exponential growth and then connect that growth to expressions involving exponents. It illustrates well how fast exponential expressions grow. Students who are just learning about exponents may need more structure than the follow-up questions provided in order to write the expressions.

Some more good follow-up questions would be, "On what day does the amount from

the magic coins become more than 50,000? On what day does the amount from the magic coins become more than 1,000,000?"

The Standards for Mathematical Practice focus on the nature of the learning experience by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, only one practice connection will be discussed in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This task connects repeated calculations with an expression involving exponents to create a shorthand notation that can be used to answer the questions being asked (MP.8). The teacher could pose this task with small or large groups and lead them through creating a table to demonstrate the repeated calculations necessary to solve this problem. Once the pattern has been discussed, the teacher could have students begin to discuss the exponential growth demonstrated by the table, solutions to the questions posed in the task and guide the students in using the repeated calculations to write an expression that could be used to efficiently solve any question that could be asked about the exponential growth.

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Solution

We can summarize the growth pattern of the coins in a table.

day	number of coins
1	2
2	4
3	8
4	
5	

To find the general pattern, we notice that to find the number of coins in a row, we doubled the number of coins in the previous row. So if we continue the pattern, at the end of day 4 there will be $2 \cdot 8 = 16$ coins, and at the end of day 5 there will be $2 \cdot 16 = 32$ coins. If we look at day 4 again, we have $16 = 2 \cdot 8$ coins. The 8 coins were obtained because $8 = 2 \cdot 4$ coins and the 4 coins came from $4 = 2 \cdot 2$ coins. So on day 4 we could also say we have $16 = 2 \cdot 8 = 2 \cdot (2 \cdot 4) = 2 \cdot (2 \cdot (2 \cdot 2)) = 2^4$ coins. We can rewrite every number of coins in the table in terms of powers of 2:

day	number of coins
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$
5	$32 = 2^5$
$\vdots$	$\vdots$
28	$268,435,456 = 2^{28}$

Having found a way to easily express the number of coins there will be on a particular day, we don't need to fill in every row in the table; instead we can use a calculator to find that $2^{28} = 268,435,456$ (which is well over a million coins). Since this number is a lot bigger than 50,000, we can conclude that it is to our advantage to choose the magical coin as our reward. The last number in the table is so large that it is hard to understand just how much bigger it is than 50, 000. To illustrate this, the followup question may be helpful. By either filling the table in systematically or by trial and error we find that on day 15 there are 2^{15} or 32,768 coins. On the very next day we have 2^{16} or 65,536 coins. So it will take just over two weeks for our magical coin to outgrow the fixed reward of 50,000 coins. In just four more days (on the 20th day) the number of coins will be $2^{20} = 1,048,576$ which is already over a million. A graph helps us to visualize how fast the exponential expression grows with each day.