6.EE,RP 7.EE,RP Anna in D.C.

Alignments to Content Standards: 6.EE.A.3 6.EE.B.7 7.RP.A.3 7.EE.B.3 6.RP.A.3.c

Task

Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of $27.50 for dinner. What is the cost of her dinner without tax or tip?

IM Commentary

The purpose of this task is to give students an opportunity to solve a multi-step percentage problem that can be approached in many ways. Because the tax and tip in this problem are a fixed percentage of the cost of the meal, the total amount paid is in a fixed ratio with the cost of the meal. The task can illustrate multiple standards depending on the tools students have already developed and the way the approach the problem. Some students might use a ratio table; others might divide by the appropriate unit rate. It can also be solved by setting up an equation with x representing the cost of the meal. Such an algebraic approach is challenging but within reach of 6th graders as the third solution shows. However, it would be best to give it to 6th graders in an instructional setting as problem-solving exercise because sixth graders will still be working to integrate these different ideas.

This task was adapted from problem #5 on the 2012 American Mathematics Competition (AMC) 10B Test. The responses to the multiple choice answers for the problem had the following distribution:

Choice	Answer	Percentage of Answers
(A)	$18	1.86
(B)	$20	4.05
(C)	$21	13.42
(D)*	$22	69.66
(E)	$24	4.27
Omit	--	6.73

Of the 35,086 students who participated, 17,169 or 49% were in 10th grade, 9,928 or 28% were in 9th grade, and the remainder were below than 9th grade.

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, the commentary will spotlight one practice connection in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This task spotlights the process of students making sense of quantities and their relationships in problem situations. The students decontextualize the task, select an approach to the problem and then manipulate the symbols used in the problem to solve the task. (MP.2) As students decontextualize the task into algebraic notation they will consider whether the order of multiplying the tax and tip makes a difference, and how to denote in symbols as part of their expression or equation the fact that the tip was taken on just the food amount rather than the amount including the tax.

Students may be given this task and asked to collaborate in small groups to solve the question(s) posed. The teacher might direct the collaboration by asking questions such as: “What do the numbers used in the task represent?” “What strategies or tools might we use to solve this problem?”(MP.5) “What properties might we use to help us find a

solution?" "What operations will we use to solve this task?" After groups have had an opportunity to work through the task and share their results, the teacher could then assign similar tasks with different unknowns and have the groups work on them as well. It is important that students just don't know how to compute the numbers in the tasks, but make a connection between the numbers and calculations and the meaning of the quantities, and are flexible in their use of operations and their properties. (MP.2)

Solutions

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Solution: 1 A Ratio Table (7.RP.3)

The sales tax of 10% combined with the tip of 15% add 25% to the bill. So each dollar the meal cost will contribute $1.25 to the bill. Below is a table showing different costs of the meal along with the total price including sales tax and tip:

1	1.25
5	6.25
10	12.50
15	18.75
20	25
25	31.25

This shows that the cost of the meal is between $20 and $25 so we can try $1 increments here:

21	26.25

22	27.50

This shows that the cost of the meal was $22.

Notice that the method employed here will require patience if the cost of the meal is not an even number of dollars. First we would find which two whole dollar amounts it is between and then would have to start a new table with increments in ten cents and, eventually, cents if necessary.

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Solution: 2 Reasoning with division (7.EE.3)

Anna paid $27.50 for the meal, the tax, and the tip. The tax is 10% of the price of the meal and the tip is 15% of the price of the meal. Combining the meal, the tax, and the tip we get 125% of the cost of the meal. Since Anna paid $27.50 total this means that 1.25 times the cost of the meal is $27.50. So dividing $27.50 by 1.25 gives the cost of the meal:

$$\frac{\$27.50}{1.25} = \$22.$$

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Solution: 3 Using algebraic expressions (6.RP.3c, 6.EE.3, 6.EE.7)

If x equals the cost of the meal without tax and tip, then the tax is 10 hundredths that amount:

$$\frac{10}{100} \cdot x = 0.10x$$

and the tip is 15 hundredths that amount:

$$\frac{15}{100} \cdot x = 0.15x$$

Using the distributive property, we can see that the total cost for the meal with tax and

tip is

$$x + 0.10x + 0.15x = (1 + 0.10 + 0.15)x = 1.25x$$

Since the total cost of the meal is $27.50, we know that is

$$1.25x = 27.50$$

and that

$$x = \frac{27.50}{1.25} = 22$$

So the cost of the meal without tax and tip is $22.