6.EE Watch out for Parentheses

Alignments to Content Standards: 6.EE.A

Task

Evaluate the following numerical expressions.
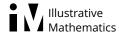
a. $2(5 + (3)(2) + 4)$

b. $2((5 + 3)(2 + 4))$

c. $2(5 + 3(2 + 4))$

Can the parentheses in any of these expressions be removed without changing the value the expression?

IM Commentary

This problem asks the student to evaluate three numerical expressions that contain the same integers yet have differing results due to placement of parentheses. Students from all levels of mathematics often make an error on (c) by first doing the operation of $5 + 3$. Parts (a) and (b) help the student recognize this error.

This type of problem helps students to see structure in expressions and represents an important transition from the work that students do in elementary school with numeric expressions to the work they will be doing in middle school with algebraic expressions. Note that this task was originally written to illustrate 5.OA.1, but Heather Brown, Illinois State Board of Eduction Math Content Specialist, pointed out that the K–5 Operations and Algebraic Thinking Progression explicitly excludes numeric expressions with nested parentheses in 5th grade. Thus, this task would be appropriate for 5th graders needing

an extra challenge or 6th graders getting ready to make the transition to working with expressions that contain variables.

Edit this solution

Solution

a. $2(5 + (3)(2) + 4)$. We may evaluate this expression in two ways:

Distributing the lead constant first:

$$2 \cdot 5 + 2 \cdot 3 \cdot 2 + 2 \cdot 4 = 10 + 12 + 8 = 30$$

or distributing the lead constant last:

$$2(5 + 6 + 4) = 2 \cdot 15 = 30.$$

Either way, we first have to multiply $(3)(2) = 6$ before adding any of the terms. The parentheses in the middle are not necessary. Instead of writing $(3)(2)$ we can say $3 \cdot 2$.

b. Notice that in the expression $2((5 + 3)(2 + 4))$, the outer set of parentheses are not necessary:

$$2((5 + 3)(2 + 4)) = 2(5 + 3)(2 + 4).$$

The other parentheses are necessary since they indicate that we first have to perform the additions inside these parentheses:

$$2(5 + 3)(2 + 4) = 2(8)(6) = 96.$$

c. In this expression we complete the operations from the inside out. The inner most addition must occur first, then the inner multiplication, then the secondary addition and finally the outer multiplication:

$$2(5 + 3(2 + 4)) = 2(5 + 3(6)) = 2(5 + 18) = 2(23) = 46.$$

In this expression all parentheses are needed.