6.EE,NS,RP; 8.EE,F Pennies to heaven

Alignments to Content Standards: 6.EE.B.6 6.NS.B.3 6.RP.A.3 8.EE.A.3 8.EE.A.4 8.F.A.1

Task

A penny is about $\frac{1}{16}$ of an inch thick.

a. In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

b. In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

c. The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?

IM Commentary

This task can be made more hands-on by asking the students to determine about how many pennies are needed to make a stack one inch high. In addition, for part (b) students could be invited to research this question. Useful information which would need to be compiled to get total mintage figures can be found at Annual mintage of Lincoln pennies. From 1909 through 2009 the total number of pennies minted is 455,627,740,918 according to Total mintage of Lincoln pennies.

The goal of this task is to give students a context to investigate large numbers and measurements. Students need to fluently convert units with very large numbers in order to successfully complete this task. The total number of pennies minted either in a single year or for the last century is phenomenally large and difficult to grasp. One way to assess how large this number is would be to consider how far all of these pennies would reach if we were able to stack them one on top of another: this is another phenomenally large number but just *how* large may well come as a surprise. In particular, before they start working on the problem, the teacher may wish to ask the students whether or not they think it would be possible to reach the moon with this giant stack of pennies.

Four solutions are offered to the problem stressing different mathematical ideas:

a. A division approach taking the distance from the earth to the moon and dividing by the height of a single penny.

b. An algebraic approach where x denotes the number of pennies necessary to reach the moon.

c. A ratio approach which could be solved algebraically, linking to solution (a), or through a ratio table.

d. Arithmetic with scientific notation.

The first of these solutions uses multiplication as well as division because there is conversion of units involved. Note that the second of these solutions leads naturally to the introduction of a function (representing the height of a stack of x pennies) and so, although it also strongly stresses ratio language, it would need to be adapted in order to be appropriate at the sixth grade level. As written, the answer to part (c) meets the 6-RP.3 standard. Students should eventually be comfortable with all three approaches. Note too that throughout the teacher needs to provide some guidance in terms of the level of accuracy with which results should be recorded: because the number of pennies minted is an estimate and the thickness of the pennies is an estimate, no more than one or two significant digits should be recorded in the answers; this is important because it makes each successive calculation easier if numbers are rounded.

The number of pennies made in the last 100 years is about 100 times the number of pennies made in 2011. The vast majority of these pennies, however, have been made in the last 40 years. In 1922, for example, only a little over 7 million pennies were made while in 1982 the total exceeded a staggering 17 billion.

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, the commentary will spotlight one practice connection in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This task helps illustrate Mathematical Practice Standard 4, Model with mathematics. There are 3 parts to this task and each part provides scaffolding opportunities for students as they apply the mathematics they know to find a solution pathway. Because this task has multiple entry points, students may apply several different approaches depending upon their own understanding, such as, arithmetical, algebraic, ratio reasoning, or scientific notation. Students may make assumptions and approximations while solving this task as the number of pennies minted and the thickness of each penny are estimates. As a result, the level of accuracy of results must be resolved which connects to Mathematical Practice 6, Attend to precision.

Solutions

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Solution: 1 Arithmetic (6-NS.2 and 6-NS.3)

a. Five billion pennies, each $\frac{1}{16}$ of an inch thick, will make a stack

$$5,000,000,000 \times \frac{1}{16} \text{ inches}$$

high. Dividing $5,000,000,000$ by 16 gives a little more than $300,000,000$ inches. We need to convert this to miles so we can begin by finding how many feet there are in $300,000,000$ inches. This will be

$$300,000,000 \div 12 = 25,000,000.$$

Finally, to find out how many miles there are in $25,000,000$ feet we need to divide by 5280 because there are 5280 feet in each mile:

$$25,000,000 \div 5280 \approx 5000.$$

So the stack of pennies would be about 5000 miles high.

b. A stack of 500 billion pennies is the same as 100 stacks of 5 billion pennies so using the answer from part (a), the stack of 500 billion pennies would be about $100 \times 5000 = 500,000$ miles high.

c. Here we need to perform the division $239,000 \ miles \div \frac{1}{16} inches$. Because the units, miles and inches, are different we need to find out how many inches are in each mile (or equivalently, convert the $239,000$ miles to inches). We have

$$\begin{aligned} 239,000 \ \text{ miles} &= 239,000 \ \text{miles} \times \frac{5280 \ \text{feet}}{\text{mile}} \times \frac{12 \ \text{inches}}{\text{foot}} \\ &= 239,000 \times 5280 \times 12 \ \text{inches} \end{aligned}$$

Multiplying this out gives $15,143,040,000$ or about 15 billion inches. Now we can calculate

$$15,000,000,000 \div \frac{1}{16} = 240,000,000,000.$$

So it takes about 240 billion pennies to make a stack high enough to reach the moon.

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Solution: 2 Algebra (6-EE.6)

a. Since it takes 16 pennies to make a stack one inch high, with $5,000,000,000$ pennies we could make a stack

$$\frac{5,000,000,000}{16} \text{inches}$$

high. The question asks how many miles this is so we need to convert inches to miles. For this, we use the facts that there are 12 inches in a foot and 5280 feet in a mile:

$$\begin{aligned}\frac{5,000,000,000}{16}\text{inches} &= \frac{5,000,000,000}{16\times 12}\text{inches}\times\frac{\text{foot}}{\text{inches}}\\ &= \frac{5,000,000,000}{16\times 12\times 5280}\text{inches}\times\frac{\text{foot}}{\text{inches}}\times\frac{\text{mile}}{\text{feet}}\\ &\approx 5000\ \text{miles.}\end{aligned}$$

Note that the exact value of the quotient above is a little more than 4932 miles but because $5,000,000,000$ is an estimate for the number the number of pennies minted in 2011 it is apprpriate to list the height of the stack as about 5000 miles.

b. A stack of $500,000,000,000$ pennies would reach 100 times as far as a stack of $5,000,000,000$ pennies. So this would be about 100×5000 miles or about $500,000$ miles.

c. If we write x for the number of pennies in a stack which reaches the moon then we have the equation

$$x\times\frac{1}{16}\text{inch} = 239,000\ \text{miles.}$$

We can solve this equation as follows:

$$x\ \text{ inches} = 16\times 239,000\ \text{miles}\times\frac{5280\ \text{ feet}}{\text{mile}}\times\frac{12\ \text{ inches}}{\text{foot}}$$

Simplifying we find

$$x\ \text{ inches} = 16\times 239,000\times 5280\times 12\ \text{ inches}$$

So x is $16\times 239,000\times 5280\times 12\approx 240,000,000,000$. So it takes about 240 billion pennies (or almost a quarter of a trillion!) to reach the moon.

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Solution: 3 Ratio (6-RP.3, 8-F.1)

Since the pennies being stacked all have the same thickness, the ratio of the number of pennies in a stack to the height of the stack does not depend on how many pennies are in the stack. In other words if a denotes the number of pennies in a stack and h is the function so that $h(a)$ denotes the height of that stack then the ratios $(a : h(a))$ are equivalent for any positive number a of pennies.

a. Here we have $a = 5,000,000,000$. According to the previous paragraph we need to find $h(5,000,000)$ we need to solve the equation

$$\left(1 : \frac{1}{16}\text{inch}\right) = (5,000,000,000 : x \text{ miles}).$$

We have

$$\left(1 : \frac{1}{16}\text{inch}\right) = \left(5,000,000,000 : 5,000,000,000 \times \frac{1}{16} \text{ inches}\right).$$

So to solve for x we need to convert $\frac{5{,}000{,}000{,}000}{16}$inches to miles which can be done as in the previous solution:

$$\begin{aligned}\frac{5,000,000,000}{16}\text{inches} &= \frac{5,000,000,000}{16 \times 12}\text{inches} \times \frac{\text{foot}}{\text{inches}} \\ &= \frac{5,000,000,000}{16 \times 12 \times 5280}\text{inches} \times \frac{\text{foot}}{\text{inches}} \times \frac{\text{mile}}{\text{feet}} \\ &\approx 5000 \text{ miles.}\end{aligned}$$

So x, the height of the stack of $5,000,000,000$ pennies, is about 5000 miles.

b. If we multiply the number of pennies in a stack by 10, the height of the stack is also multiplied by 10 (so that the ratio is the same). Since $h(500,000,000,000) = 10h(50,000,000,000)$ and $h(50,000,000,000)$ is about 5000 miles, we have that $h(500,000,000,000) \approx 500,000$ miles.

c. We need to find x to solve the following equation of ratios:

$$\left(1 : \frac{1}{16}\text{inch}\right) = (x : 239,000 \text{ miles}).$$

We can first convert miles to inches as in part (a):

$$239,000 \text{ miles} = 12 \times 5280 \times 239,000 \ \frac{\text{inches}}{\text{foot}} \times \frac{\text{feet}}{\text{mile}} \times \text{miles.}$$

Putting this into the previous equation gives

$$\left(1 : \frac{1}{16}\text{inch}\right) = (x : 12 \times 5280 \times 239,000 \text{ inches}).$$

Solving this gives

$$x = 16 \times 12 \times 5280 \times 239,000 \approx 240,000,000,000.$$

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Solution: 4 Scientific notation (8-EE.3 and 8-EE.4)

Scientific notation is appropriate for this problem as the number of pennies involved is very large and nicely represented using exponential notation. Either of the above solutions can be adapted for the use of exponential notation and we have chosen here to use the first method.

a. One billion is one with 9 zeroes or 1×10^9. So 5 billion pennies is 5×10^9 pennies. Since it takes 16 pennies to make a stack one inch high, with 5×10^9 pennies we could make a stack

$$\frac{5 \times 10^9}{16} \text{inches}$$

high. The question asks how many miles this is so we need to convert inches to miles. For this, we use the facts that there are 12 inches in a foot and 5280 feet in a mile:

$$\begin{aligned}
\frac{5 \times 10^9}{16} \text{inches} &= \frac{5 \times 10^9}{16 \times 12} \text{inches} \times \frac{\text{foot}}{\text{inches}} \\
&= \frac{5 \times 10^9}{16 \times 12 \times 5280} \text{inches} \times \frac{\text{foot}}{\text{inches}} \times \frac{\text{mile}}{\text{feet}} \\
&\approx \frac{5 \times 10^9}{1 \times 10^6} \text{ miles} \\
&= 5 \times 10^3 \text{ miles.}
\end{aligned}$$

In the last step, we use the law of exponents

$$\frac{10^9}{10^6} = 10^{9-6}.$$

b. A stack of $500,000,000,000$ pennies is 5×10^{11} pennies or 100 times as many as the stack of 5×10^9 considered in part (a). So this stack would reach 100 times as far as a stack of 5×10^9 pennies or about

$$100 \times 5 \times 10^3 = 5 \times 10^5 \text{ miles.}$$

c. If we write x for the number of pennies in a stack which reaches the moon then we have the equation

$$x \times \frac{1}{16}\text{inch} = 239,000 \text{ miles}.$$

We can solve this equation as follows:

$$x \text{ inches} = 16 \times 239,000 \text{ miles} \times \frac{5280 \text{ feet}}{\text{mile}} \times \frac{12 \text{ inches}}{\text{foot}}$$

Simplifying we find

$$x \text{ inches} = 16 \times 239,000 \times 5280 \times 12 \text{ inches}$$

So x is $16 \times 239,000 \times 5280 \times 12 \approx 2.4 \times 10^{11}$. So it takes about 2.4×10^{11} pennies (or almost a quarter of a trillion!) to reach the moon.