6.RP Bag of Marbles

Alignments to Content Standards: 6.RP.A.3.a 6.RP.A.1

Task

In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

IM Commentary

The purpose of this task is to help students develop fluency in their understanding of the relationship between fractions and ratios. It provides an opportunity to translate from fractions to ratios and then back again to fractions.

One interesting aspect to the two solutions provided is that a variable x is introduced in the second solution but not in the first. The reason for the variable x is that we are given that for each group of 3 blue marbles there are 4 red marbles. But we don't know how many of these groups there are and x is a convenient symbol to represent this number. In the first solution, the phrase "4 out of every 7 marbles in the bag are red" conceals the variable x since another way of saying this would be that there are $4x$ red marbles and $7x$ marbles total.

This task was adapted from problem #7 on the 2012 American Mathematics Competition (AMC) 10A Test. For the 2012 AMC 10A, which was taken by 73073 students, the multiple choice answers for the problem had the following distribution:

Choice	Answer	Percentage of Answers

(A)	$\frac{2}{5}$	1.92
(B)	$\frac{3}{7}$	3.03
(C)*	$\frac{4}{7}$	74.86
(D)	$\frac{3}{5}$	1.32
(E)	$\frac{4}{5}$	15.74
Omit	--	3.11

Of the 73703 students: 36206, or 49%, were in 10th grade;25498 or 35%, were in 9th grade; and the remainder were below than 9th grade.

Solutions

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Solution: 1 Reasoning with ratios (6.RP.1)

We are given that $\frac{3}{5}$ of the marbles in the bag are blue and the rest are red. This means that $\frac{2}{5}$ of the marbles in the bag are red. We are not given how many blue and red marbles there are but we can see that the ratio of blue to red marbles is 3:2. If the number of red marbles is doubled then the new ratio will be 3:4. To see this, note that initially, for every 3 blue marbles there are 2 red marbles. Doubling the red marbles means that each 2 red marbles will be replaced by 4 red marbles. So after doubling the red marbles for each 3 blue marbles there will now be 4 red marbles, giving a new ratio of 3:4.

With our new ratio of 3:4 for blue marbles to red marbles, this means that 4 out of every 7 marbles in the bag are red. So $\frac{4}{7}$ of the marbles are now red.

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Solution: 2 Making a table (6.RP.3a, 6.EE.7)

We are given that for every three blue marbles in the bag, there are two red marbles. Below is a table indicating some possibilities for the number of blue and red marbles in the bag and the associated ratios:

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Blue Marbles	Red Marbles	Ratio of Blue to Red Marbles
3	2	3:2
6	4	6:4
15	10	15:10
30	20	30:20
3x	$2x$	$3x$:$2x$

Notice that whatever whole number we choose for x, all of the ratios $3x$:$2x$ are equivalent. Now we make another table, doubling the number of red marbles:

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Blue Marbles	Red Marbles	Ratio of Blue to Red Marbles
3	4	3:4
6	8	6:8
15	20	15:20
30	40	30:40
3x	4x	$3x$:$4x$

These ratios are also all equivalent. To find the fraction of red marbles in the bag, we work with the ratio $3x$:$4x$ where x is some whole number. So there are $3x$ blue marbles and $4x$ red marbles in the bag. This means that there are $3x + 4x = 7x$ marbles total, $4x$ of which are red. The fraction of red marbles in the bag is therefore $\frac{4x}{7x} = \frac{4}{7}$. It is important in this final step, canceling x's, to know that x is not zero. We do know this because $x = 0$ would mean that there are no marbles in the bag.