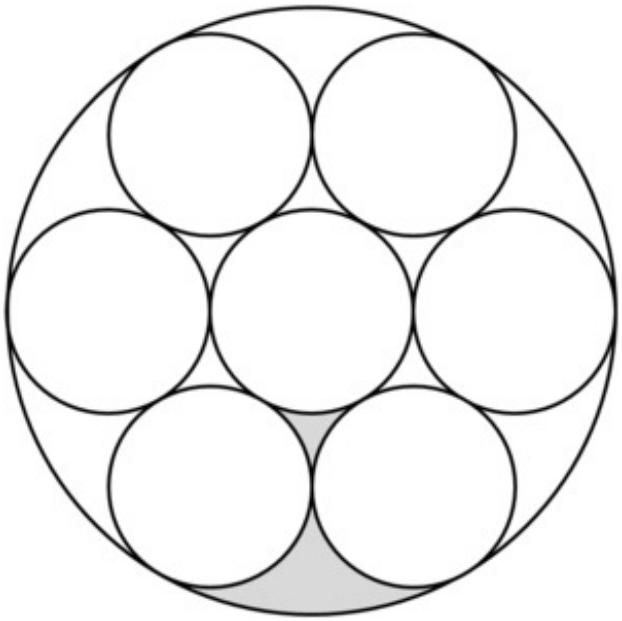
7.G Eight Circles

Alignments to Content Standards: 7.G.B.4

Task

The figure below is composed of eight circles, seven small circles and one large circle containing them all. Neighboring circles only share one point, and two regions between the smaller circles have been shaded. Each small circle has a radius of 5 cm.



Calculate:

a. The area of the large circle.

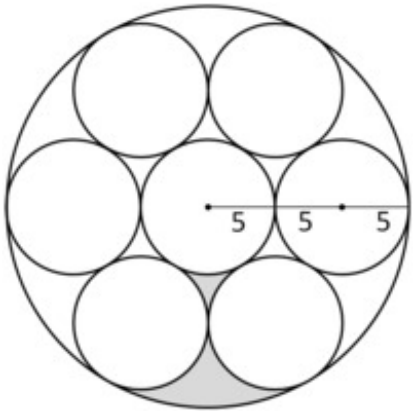
b. The area of the shaded part of the figure.

IM Commentary

The purpose of this task is to strengthen students' understanding of area. It could be assigned in class to individuals or small groups or given as a homework exercise to generate interesting discussions the following day. The relatively high levels of complexity and technical demand enhance its instructional value. Seventh grade student may require some teacher guidance to be successful with this task.

Edit this solution

Solution



The radius of each of the seven small circles is 5 cm. This makes the radius of the large circle $3 \cdot 5 = 15$ cm.

$$\text{Area of a Circle} = \pi r^2$$

The area of the large circle is $\pi(15 \text{ cm})^2 = 225\pi \text{ cm}^2$.

The area of each small circle is $\pi(5 \text{ cm})^2 = 25\pi \text{ cm}^2$.

There are seven small circles in all, so the area of all the small circles together is

$$7 \times 25\pi \text{ cm}^2 = 175\pi \text{ cm}^2.$$

If we take the area of the large circle and subtract the area of the seven small circles, we will be left with all of the area contained in the large circle that is not contained in a

small circle, that is, the area around the small circles. This area is $225\pi \text{ cm}^2 - 175\pi \text{ cm}^2 = 50\pi \text{ cm}^2$. Notice that the exact shape of the shaded region is repeated six times in the large circle. This makes the shaded region $\frac{1}{6}$ of the area that is contained in the large circle that is not contained in a small circle. Thus the shaded region has an area of

$$\frac{1}{6} \times 50\pi \text{ cm}^2 = \frac{50\pi}{6} \text{ cm}^2 = \frac{25\pi}{3} \text{ cm}^2.$$