7.G,RP Measuring the area of a circle

Alignments to Content Standards: 7.G.B.4 7.RP.A.3

Task

Juan wants to know the cross-sectional area of a circular pipe. He measures the diameter which he finds, to the nearest millimeter, to be 5 centimeters.

a. How large is the possible error in Juan's measurement of the diameter of the circle? Explain.

b. As a percentage of the diameter, how large is the possible error in Juan's measurement?

c. To find the area of the circle, Juan uses the formula $A = \pi r^2$ where A is the area of the circle and r is its radius. He uses 3.14 for π. What value does Juan get for the area of the circle?

d. As a percentage, how large is the possible error in Juan's measurement for the area of the circle?

IM Commentary

This goal of this task is to give students familiarity using the formula for the area of a circle while also addressing measurement error and addresses both 7.G.4 and 7.RP.3.

Students might find it confusing to be asked to find the area of the cross-section of the pipe since looking at the cross section, it is "empty" in the middle. This confusion can even arise when we just ask students to find the area of a circle drawn in the Euclidean plane. It is common for students to get confused about which parts of the figure are part of the circle itself (i.e. the curve that represents the set of points equidistant from the center) and which are just defined by the circle (i.e. the region inside it). One reason this can be confusing to students is that we ask them to "Find the circumference and area of the circle," when in fact we mean, "Find the circumference of the circle and the area of the region it encloses." In the Common Core, students start studying area and perimeter in 3rd grade, and should be grappling with this distinction then; see http://www.illustrativemathematics.org/illustrations/1514. Even so, students may need some support to understand what is meant by finding the cross-sectional area of a hollow pipe.

It would be good for students to make their own measurements of a circular object before tackling this task (ideally the cross-section of a pipe, although the context of the task can be modified to fit whatever circular objects the students are measuring). Once they have made their own measurements and calculations, they can share results and analyze the possible sources of error.

When students have completed the task and are confident about their solutions, the teacher should highlight the fact that possible errors in measurement generally become worse when quantities are multiplied. This can be contrasted with what happens when quantities are added: if Juan were to make two linear measurements, each accurate to within one percent, then their sum would also be accurate to within one percent.

Further aggravating the inaccuracy in this calculation is the fact that 3.14 is only an approximate value for π. This is a different type of error, however, because more decimal places of π could be used if desired. In part (d) of the problem Juan's value of 19.625 square centimeters is not halfway between the values of 19.244 square centimeters and 20.030 square centimeters: it is closer to 19.244 square centimeters. Using a more accurate value for π gives a value of about 19.635 square centimeters which is closer to halfway between the values of 19.244 square centimeters and 20.030 square centimeters.

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Solution

a. Juan finds that the circle is 5 cm or 50 mm in diameter. Since the tape measure is accurate to the nearest millimeter, this means that the actual diameter is between 4.95 centimeters and 5.05 centimeters.

This is so because if the circle were less than 4.95 centimeters in diameter then it would be closer to 4.9 centimeters (or 49 millimeters) than to 5 centimeters (or 50 millimeters). Similarly, if the circle were more than 5.05 centimeters in diameter it would be closer to 51 millimeters than to 50 millimeters. This means that the diameter of the circle is within 0.05 centimeters or 0.5 millimeters of Juan's measurement.

b. We can find what percent the greatest possible error is of the measurement by dividing the possible error (0.05 cm) by the measurement (5 cm). Since

$$0.05 \div 5 = 0.01,$$

the possible error in Juan's measurement is one percent of the diameter.

c. Juan's estimate for the diameter of the circle is 5 centimeters. This means that his estimate for the radius is 2.5 centimeters. So Juan finds

$$3.14 \times 2.5^2 = 19.625$$

square centimeters as an estimate for the area of the circle.

d. From part (a) the largest the diameter could be is 5.05 centimeters and so the largest the radius could be is half of this or 2.525 centimeters. So the largest the area could be is

$$\pi \times 2.525^2 \approx 20.030$$

square centimeters. Juan's value of 19.625 square centimeters differs from this by 0.405 or just over 2 percent.

The smallest the diameter could be is 4.95 centimeters and so the smallest the radius could be is half of this or 2.475 centimeters. So the smallest the area could be is

$$\pi \times 2.475^2 \approx 19.244$$

square centimeters. Juan's value of 19.625 square centimeters differs from the smallest possible area by 0.381 square centimeters and from the largest possible area by 0.405 square centimeters.

To find the percent of possible error in Juan's area estimate, first note that his estimate is further from the largest possible area, 20.30 square centimeters, than from the smallest possible area, 19.244 square centimeters. So to find the percent of error in the worst case we need to calculate what percent 0.405 square centimeters is of 20.30 square centimeters. We first find the fraction

$$\frac{0.405 \text{ square centimeters}}{20.30 \text{ square centimeters}} \approx 0.02.$$

Since one percent corresponds to 0.01 this means that the possible error in Juan's measurement is about 2 percent.

If Juan knows that his measurement has a possible error of 2 percent, or about 0.4 square centimeters, then a good way to record his answer would be 19.6 ± 0.4 square centimeters. (Given that the measurement of the diameter wasn't more accurate than the nearest millimeter, it's not appropriate to report more decimal places than this.)