Illustrative Mathematics

7.G Drinking the Lake

Alignments to Content Standards: 7.G.B

Task

a. About how much water does an average person drink in a life time? Explain.

b. What size container would you need to hold this amount of water?

c. In a life time, could a person drink all of the water in a lake, such as Crater Lake pictured below?

IM Commentary

The purpose of this task is for students to solve a volume problem in a modeling context (MP.4). It requires multiple conversions using units of time and volume. It also requires thinking about some reasonable hypotheses for how much an individual drinks each day and also how many days there are in that individual's lifetime. Many different, reasonable answers are possible. The teacher may wish to have a class discussion in order to choose uniform values for these numbers, but it is also appropriate to have students work in groups and then each group can choose values and then discuss and justify them as a whole group.

Students first study volume in 5th grade (5.MD.C.3) and start symbolic manipulations for the purpose of unit conversions in 6th grade (6.RP.A.3.c). In 5th and 6th grade they also solve volume problems involving right rectangular prisms (5.G.C.3, 6.G.A.2), but it isn't until 7th grade that they are asked to solve more complex volume-related problems. This task asks students to work with volumes that do not have a well-defined shape, and so is more abstract than it appears. The fact that the volume of a liquid stays the same regardless of the shape of the container it is in is not obvious and depends on knowledge of the physical world (namely, that liquids are incompressible). While the standards do not specifically address when students should be able to solve such problems, 7th grade seems appropriate. That is when students are using the fact that the volume of a figure can be found by decomposing it into non-overlapping pieces, finding the volume of the pieces, and then adding their volumes (as implied by 7.G.B.6), indicating that they are developing a more abstract understanding of volume than is called for in 5th and 6th grade.

One advantage to the metric system of measurement is that the physical properties of water connect different units: one liter of water has a mass of 1 kilogram and it takes up 1000 cubic centimeters of space. If we work in the U.S. Customary System, then the liquid measures of cups, pints, quarts, and gallons require an extra conversion in order to be related to other volume units such as cubic inches or cubic feet. In order to convert liquid measures into volume units, we can convert gallons to liters, liters to cubic centimeters, and cubic centimeters to cubic inches. The third solution makes these conversions while the second solution remains within the U.S. Customary System, making an estimate of how much spae a gallon of water occupies. The first solution works in metric units throughout.

Some information about the old adage to drink 8 glasses of water a day (the quantity used in one solution) is available here: http://www.mayoclinic.com/health/water/NU00283. Much more information about Crater Lake can be found here: http://en.wikipedia.org/wiki/Crater_Lake. Included is an estimate for the amount of water in Crater Lake, 18.7 cubic *kilometers.* While question (c) definitely has a negative answer, an engaging follow up to this question would be whether or not all of the people in a city (or country) could drink the water in Crater

Lake in their life times. According to the estimates of the first solution, an average person drinks only 60 cubic meters of water in a lifetime: since there are 1000 × 1000 × 1000 cubic meters in a cubic kilometer, this means that it would take about 300,000,000 people to consume all of the water in Crater Lake, throughout their lifetimes. So there is almost enough water in Crater Lake for the entire population of the United States!!

This task is based upon problem 1 in Class Activity 11K of Sybilla Beckmann's *Mathematics for Elementary School Teachers,* Pearson, 2011. Ideas from the commentary and solution have been gathered from a fall 2013 class of prospective teachers at the University of New Mexico.

Solutions

Edit this solution

Solution: 1 Using Liters

a. A common recommendation is that we should make sure to drink at least 8 glasses of water each day. There are 4 glasses in a quart so this is the same as two quarts of water. A quart is very close to a liter so we will assume that the average person drinks 2 liters of water each day.

Next we need to determine how many days there are in the person's lifetime. If we take 80 years for an average lifetime, then there are 365 days in each year so this gives

$$80 \times 365 = 29,200 \text{ days.}$$

The number is only approximate so 30,000 days would be a good round estimate. If the average person drinks 2 liters of water each day and lives for about 30,000 days this will be

$$30,000 \times 2 = 60,000$$

liters of water consumed in a lifetime.
<./li>

b. A liter of water occupies 1000 cubic centimeters of space so 60,000 liters of water will take up

$$60,000 \times 1,000 \text{ cm}^3 = 60,000,000 \text{ cm}^3.$$

To see how much space this would take up, we can think of the 1,000 cubic centimeters as a cube which is 10 cm by 10 cm by 10 cm. One thousand of these cubes could be placed in 10 layers with each layer containing 10 rows and 10 columns. So one 1000 of the one liter cubes would take up a space 10 times as wide, 10 times as long, and 10 times as high. Since 10 × 10 cm = 100 cm, this means that a cube one meter wide, one meter long, and one meter high would hold 1,000 liters of water. To get 60,000 liters of water we can increase the width by a factor of 5, the length by a factor of 6 and the height by a factor of 2. So this means that a 5 meter by 6 meter by 2 meter container could hold 60 of the 1 meter cubes and 60,000 liters of water. This container would be about the size of a large room.

c. Crater Lake is a large lake in Oregon known for the exceptional color of its glacier water and its great depth. The trees on the island give an idea of how large the lake is. This would be thousands of times more water than we calculated above so even allowing for different estimates, it is not possible that a person could drink all of the water in Crater Lake in a life time. In fact, a person could not drink all of the water in a large pond according to the calculations in part (b): 5 meters by 6 meters by 2 meters is the size of a small swimming pool.

Edit this solution

Solution: 2 Using U.S. Customary Units

a. Rather than the proverbial 8 glasses of water a day, here we will work with a larger estimate of 1 gallon a day. There are 4 quarts in a gallon, two pints in a quart, and 2 cups in a pint so there are 4 × 2 × 2 = 16 cups of water in a gallon: this is twice as much water as we considered in the first solution.

Next we need to determine how many days there are in the person's lifetime. If we take 80 years for an average lifetime, then there are 365 days in each year so this gives

$$80 \times 365 = 29,200 \text{ days.}$$

The number is only approxomate so 30,000 days would be a good round estimate. If the average person drinks a gallon of water each day and lives for about 30,000 days this will be 30,000 gallons of water consumed in a lifetime.

b. To find out how much space a gallon occupies, we could take approximate measurements of a gallon (or half gallon) container of water or milk for example. For a gallon container of milk, the dimensions are about 6 inches by 6 inches at the base and 8 inches to the fill line for the milk. This will give an overestimate because the container has rounded edges and it is tapered where the handle is but it should be good enough to give an idea of how much space the 30,000 gallons of water will occupy. If we convert to feet (inches are too small of a unit to get a good idea of the space 30,000 gallons of water will take up) then this is

$$\frac{1}{2}\text{ foot} \times \frac{1}{2}\text{ foot} \times \frac{2}{3}\text{ foot.}$$

Multiplying this out gives $\frac{1}{6}$ of a cubic foot. If one gallon of water takes up $\frac{1}{6}$ of a cubic foot, then 30,000 gallons of water will take up 5,000 cubic feet. This would fit in a space that is 20 feet by 25 feet by 10 feet which would be about the size of a swimming pool.

c. The lake in the picture is much larger (and also much deeper) than a swimming pool. It is not possible that a person could drink all of the water in this lake.

Edit this solution

Solution: 3 Converting U.S. customary units to metric units for part (b)

Here we work with the estimate from the second solution, 60,000 gallons of water, and find out how much space this will occupy by converting to metric units and using the fact that one liter of water occupies 1000 cubic centimeters of space.

We use the fact that one gallon is about 3.8 liters. So 30,000 gallons is

$$30,000 \times 3.8\text{ liters} \approx 110,000\text{ liters.}$$

One liter occupies 1000 cubic centimeters so 110,000 liters of water will occupy 110,000 × 1000 cubic centimeters or 110,000,000 cubic centimeters. There are 2.54 centimeters in an inch and so there are 2.54^3 cubic centimeters in a cubic inch. So our water would occupy

$$110,000,000 \div 2.54^3\text{ cubic inches} \approx 7,000,000\text{ cubic inches.}$$

There are 12 inches in a foot and so

$$\begin{aligned} 7,000,000\text{ cubic inches} &= 7,000,000\text{ cubic inches} \times \frac{1}{12^3}\frac{\text{cubic foot}}{\text{cubic inches}} \\ &\approx 4000\text{ cubic feet.} \end{aligned}$$

Now 4000 = 20 × 25 × 8 so 30,000 gallons of water would occupy an area of about 20 feet by 25 feet by 8 feet. This is perhaps the size of a classroom or a swimming pool.