7.RP Proportionality

Alignments to Content Standards: 7.RP.A.2.c

Task

A text book has the following definition for two quantities to be directly proportional:

We say that y is directly proportional to x if $y = kx$ for some constant k.

For homework, students were asked to restate the definition in their own words and to give an example for the concept. Below are some of their answers. Discuss each statement and example. Translate the statements and examples into equations to help you decide if they are correct.

- Marcus:

This means that both quantities are the same. When one increases the other increases by the same amount. An example of this would be the amount of air in a balloon and the volume of a balloon.

- Sadie:

Two quantities are proportional if one change is accompanied by a change in the other. For example the radius of a circle is proportional to the area.

- Ben:

When two quantities are directly proportional it means that if one quantity goes up by a certain percentage, the other quantity goes up by the same percentage as well. An

example could be as gas prices go up in cost, food prices go up in cost.

- Jessica:

When two quantities are proportional, it means that as one quantity increases the other will also increase and the ratio of the quantities is the same for all values. An example could be the circumference of a circle and its diameter, the ratio of the values would equal π.

IM Commentary

The task has two main purposes. (1) Students make sense out of the definition of direct proportionality. (2) They engage in SMP 3 "Make a viable argument and critique the reasoning of others" and SMP 6 "Attend to precision".

Being asked to read other people's explanations of a definition forces the students to engage with the definition at a deeper level. To help students decide if an explanation is correct, teachers could encourage them to translate the words into equations and to try to "break" the rewordings. This leads naturally into attending to precision of language. Many of the given explanations are partially correct but not precise enough. The explanation "Two quantities are proportional if one change is accompanied by a change in the other." is true but incomplete. The explanation "When two quantities are directly proportional it means that if one quantity goes up by a certain percentage, the other quantity goes up by the same percentage as well." is actually correct, but not obvious.

Students also have to decide if a given example illustrates the definition. Do do this, students should again be encouraged to translate the words into equations.

Edit this solution

Solution

- Marcus

The two quantities do not have to be the same to be proportional. If we say they are the same, this means that $y = x$, which is only one example of proportionality, but

there are infinitely many more. The second sentence is also incorrect, if x increases by some amount, y does not have to increase by the same amount. For example, if $y = 2x$, then if x increases by 1, y will increase by 2.

The example is not informative. The amount of air in a balloon is usually measured by its volume.

- Sadie

Sadie's definition is not precise enough. Just saying that change in one quantity results in change in the other is to general and true for many relationships that are not proportional. The radius is directly proportional to the square root of the area of the circle, $r = \frac{1}{\pi}\sqrt{A}$, not $r = kA$ as Sadie says.

- Ben

Ben's definition actually works: If x increases by 10%, then for $y = kx$ we have

$$y = k(1.1x) = 1.1(kx),$$

and we see that y also increases by 10%. This also works if x changes by a factor of r then $y = k(rx) = r(kx)$, so y also changes by the same factor. The example is not stated precisely enough. It might be that gas prices and food cost are proportional to each other in certain situations, but that is not a general law or principle.

- Jessica

Jessica's definition works. She says that $k = y/x$, which is equivalent to $y = kx$. Her example also works: $C = \pi r$.