7.RP Art Class, Assessment Variation

Alignments to Content Standards: 7.RP.A.2

Task

The students in Ms. Baca's art class were mixing yellow and blue paint. She told them that two mixtures will be the same shade of green if the blue and yellow paint are in the same ratio.

The table below shows the different mixtures of paint that the students made.

Amount of Yellow Paint (cups)	0.5	1	1.5	2	3
Amount of Blue Paint (cups)	0.75	2	3	3	4.5

a. How many different shades of paint did the students make?

b. Which mixture(s) make the same shade as mixture A?

c. How many cups of yellow paint would a student add to one cup of blue paint to make a mixture that is the same shade as mixture A?

d. Let b represent the number of cups of blue paint and y represent the number of cups of yellow paint in a paint mixture. Write an equation that shows the relationship between the number of cups of yellow paint, y, and the number of cups of blue paint,

b, in mixture E.

IM Commentary

This task is part of a joint project between Student Achievement Partners and Illustrative Mathematics to develop prototype machine-scorable assessment items that test a range of mathematical knowledge and skills described in the CCSSM and begin to signal the focus and coherence of the standards.

Task Purpose

This task is part of a set of three assessment tasks for 7.RP.2.

a. 7.RP.2 Robot Races asks students to "explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation" and to "compute unit rates associated with ratios of fractions." Students also need to compare the speeds of the robots.

b. 7.RP.2 Art Class requires students to decide whether two quantities are in a proportional relationship by testing for equivalent ratios in a table, to find a unit rate for a ratio defined by non-whole numbers, and to represent a proportional relationship with an equation. Part (a) essentially asks students to partition a set of ratios displayed in a table into two sets of equivalent ratios. Part (b) asks students to identify all the ratios in the table that are equivalent to a given ratio. These two parts work together: the first question asks students to make a judgment about how many different proportional relationships are represented in the table, and the second asks students to specifically identify all of the ratios that go with one of those relationships. This task shows a shift in the standards that expand upon common approaches to "proportional reasoning" because it requires students to understand different aspects of proportional relationships, not just their ability to set up and solve a proportion.

c. 7.RP Buying Bananas requires students to find a unit rate for a ratio of non-whole numbers. Note that there are two distinct unit rates in this context, and that part (a) asks students to find one while part (b) asks students to find the other. Part (c) addresses one aspect of 7.RP.A.2.a "Decide whether two quantities are in a proportional relationship, e.g., by... graphing on a coordinate plane and observing whether the graph is a straight line through the origin." While the standard asks students to do the graphing themselves, the task asks them to recognize that that if two points lie on the same line through the origin then they represent quantities in the

same proportional relationship. This task shows a shift in the standards by directly assessing students' ability to interpret the graph of a proportional relationship, which is a representation that was infrequently used before the Common Core Standards.

Cognitive Complexity

Mathematical Content

Task 1: Students are introduced to constant speed in 6th grade but they are not asked to interpret graphs that represent objects moving at constant speed until 7th grade. The first option under Part (a) reflects a common student error where they interpret graphs as position graphs even when they aren't. Correctly interpreting the point on the graph and computing the unit rate are straight-forward applications of the mathematics described in 7.RP.A and comparing the speeds in Part (b) is only slightly more complex.

Task 2: Students must work with ratios of whole numbers and common decimals between 0-5. Ratios involving only whole numbers were introduced in the prior grade; the 7th grade expectation is that students will work with ratios of non-whole numbers. Additionally, this task addresses the transition between working with ratios in isolation to thinking of ratios as defining proportional relationships.

Task 3: Part (c) of the third task assess students' understanding of proportional relationships on two different levels. Concretely, it asks them to recognize that 13 pounds of bananas for $10.40 is proportional to 6.5 pounds of bananas for $5.20, which can be determined without thinking about the geometric representation of proportional relationships. However, for students to recognize that C and D are also in the same proportional relationship, they must be able to draw on the fact that quantities that are in a proportional relationship determine a line through the origin.

Mathematical Practice

Task 1: The first task does not assess any of the standards for mathematical practice any more than typical day-to-day mathematical work.

Task 2: The second task addresses several standards for mathematical practice. While it is possible that students have thought about what makes one paint mixture the same shade as another, it is unlikely they have thought about this from a mathematical perspective. Thus, students will need to make sense of the context and choose a mathematical approach to answer the questions given (there are multiple approaches). Most approaches require multiple steps, so students will need to make sense of the problem and persevere in solving it (MP 1). Students solving this task may look for

structure (MP 7) by converting all five ratios into unit ratios and then grouping the ratios that have the same unit ratio. Students might also find equivalent ratios with the same amount of one kind of paint or the same total amount of paint. Any solution approach requires students to decontextualize and contextualize (MP 2). The complexity of the item could be lowered by asking Part (b), Part (c), and then Part (a) because it would suggest a solution approach to Part (a). Complexity could be increased by removing Part (c) which helps students choose a solution method for Part (d).

Task 3: The third task also addresses several standards for mathematical practice. Students will have very likely been to the grocery store and bought items that cost a certain price per pound, but it is unlikely they will have seen this kind of information represented graphically outside of math class. Furthermore, the number of different lines represented in the coordinate plane will likely be an unfamiliar setup for students and there are lines that represent relationships that would be difficult to make sense in the context. This means that students will need to decide which lines do make sense and what they mean as well as which ones do not (MP 1). In order to recognize that points C and D correspond to bananas that have the same cost per pound in Part (c), students must reason abstractly and quantitatively (MP 2). This task also taps into students' attention to precision (MP 6) because students need to specify the units in Part (a) and attend carefully to the way the axes are labeled in Part (c).

Linguistic Demand

Task 1: It is difficult to come up with a realistic context that is both familiar to all 7th graders and where objects actually move at a constant speed. The context of racing solar-powered robots will not be familiar to all students, so a brief video clip showing a robot moving at a constant speed removes some of the linguistic complexity introduced by making sense of a verbal description in an unfamiliar context (especially ELL students). The language structure for this task is not very complex.

Task 2: There are three sentences with approximately 50 words. Two of the sentences are simple, and the other is conditional. The first two questions are simple while the third is more complex (29 words). The set up and question in Part D is longer than the stem (approximately 60 words). There are no unfamiliar words in the stem (mixture and represent are Grade 4 words and relate is Grade 6).

Task 3: The linguistic complexity of the first two questions is much lower than the third, which is a much more complex sentence as well as a more complex mathematical request.

Stimulus Material

Task 1: There is a verbal description of the context and a short video clip meant to decrease the linguistic complexity, although it increases the stimulus complexity. Students have to connect a verbal description of a context to a graphical representation of the relationships described. The racing setup is meant to help motivate the graphical representation of the information, although it increases the complexity in the sense that students could simply be given the graphs.

Task 2: The text describes the ideas about paint ratios and the information in the table organizes 10 amounts into the five ratios that the student must consider. There is no extraneous information in this stem.

Task 3: Students have to connect a verbal description of a context to a graphical representation of the relationship described. The distractors in the graph make it moderately complex; students typically see one or at most two graphs on the same coordinate plane. There is no extraneous information in the stem.

Response Mode

Task 1: Students are asked to "select all that apply" for the first part (a variant of the familiar multiple choice) and to choose one and fill in blanks if they select the correct one. This type of interface is not complex.

Task 2: Students will type in their answers, so it is not complex.

Task 3: Students will type in their answers for the first two and "select all that apply" for the third (a variant of the familiar multiple choice). This interface is not complex.

Additional Notes

- Task 2: While students can use the work they do in one part of the task to help them answer questions in other parts, they can also answer each question independently.
- Task 2: It would be possible to have an algorithm to produce variants of this task with different numbers and to have the mixtures appear in random order. The ratios were chosen so there are only two different shades of paint, but it would be possible to increase the number of mixtures and/or the number of distinct shades of paint.
- Task 3: The context for this task is what we call a "thin" context, which contrasts with a "phony" context. A *thin context* is a context that does not invoke all or even most of the complexity of a real-world situation but still plays a critical role in helping students make sense of the mathematics. A thin context can also provide students with an opportunity to interpret some piece of mathematics in a situation, which is a critical component skill required for more complex mathematical modeling tasks. The context

in this task falls under the second of these two uses of a thin context. By contrast, a *phony context* is a context that is both unrealistic and also plays no role in helping students understand or make sense of the mathematics.

Edit this solution

Solution

a. The students made **2** different shades of paint.

b. Mixtures **D** and **E** make the same shade as mixture A.

c. A student should add $\frac{2}{3}$ cup of yellow paint to 1 cup of blue paint to make the same shade as mixture A.

d. Either of these equations would be correct:

- $b = \frac{3}{2}y$ (or $\frac{3}{2}y = b$ if this is a fill-in-the-blank)
- $y = \frac{2}{3}b$ (or $\frac{2}{3}b = y$ if this is a fill-in-the-blank)