7.RP Two-School Dance

Alignments to Content Standards: 7.RP.A.3

Task

There are 270 students at Colfax Middle School, where the ratio of boys to girls is 5:4. There are 180 students at Winthrop Middle School, where the ratio of boys to girls is 4:5. The two schools hold a dance and all students from both schools attend. What fraction of the students at the dance are girls?

IM Commentary

The purpose of this task is to see how well students students understand and reason with ratios. A common mistake is to simply add the ratios values, i.e. getting 10:10 and then reinterpreting that to mean half of the the students at the dance are girls; this task would be appropriate for formative or summative assessment. Students must work with two different ratios in order to calculate the number of girls at Colfax and Winthrop Middle Schools. This information is then combined to find the fraction of students at the two school combined who are girls. There are multiple ways to use the given ratios to find the number of girls at each school, including arithmetic reasoning, setting up a ratio table, and using an algebraic equation to represent the proportional relationship.

This task was adapted from problem #14 on the 2011 American Mathematics Competition (AMC) 8 Test. The responses to the multiple choice answers for the problem had the following distribution:

4.75

Choice	Answer	Percentage of Answers

(A)	$\frac{7}{18}$	7.24
(B)	$\frac{7}{15}$	12.14
(C)*	$\frac{22}{45}$	36.59
(D)	$\frac{1}{2}$	30.47
(E)	$\frac{23}{45}$	8.72
Omit	-	

Of the 153,485 students who participated, 72,648 or 47% were in 8th grade, 50,433 or 33% were in 7th grade, and the remainder were less than 7th grade.

Solutions

Solution: 1 Arithmetic reasoning with ratios

At Colfax Middle School we are given that out of every 9 students 5 are boys and 4 are girls. There are $270 = 30 \times 9$ students at Colfax Middle School so $30 \times 5 = 150$ of these are boys and $30 \times 4 = 120$ of them are girls.
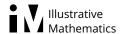
At Winthrop Middle School we are given that out of every 9 students 4 are boys and 5 are girls. There are $180 = 20 \times 9$ students at Winthrop Middle School so $20 \times 4 = 80$ of these are boys and $20 \times 5 = 100$ of them are girls.

At the two schools combined there are $150 + 80 = 230$ boys and $120 + 100 = 220$ girls. So the fraction of students at the dance who are girls is

$$\frac{220}{220 + 230} = \frac{220}{450} = \frac{22}{45}.$$

Solution: 2 Using a linear equation

The question asks what fraction of the students at the two schools are girls. At Colfax Middle School the ratio of boys to girls is 5 to 4. In other words for every of every 4 girls there are 9 total students. If we let x be the number of girls at Colfax Middle School and y the total number of students this means that

$$y = \frac{9}{4}x.$$

We are given that there are 270 students at Colfax Middle School. Setting $y = 270$ we can solve for x to find that there are $270 \times \frac{4}{9} = 120$ girls at Colfax Middle School.

At Winthrop Middle School the ratio of boys to girls is 4 to 5. In other words for every of every 5 girls there are 9 total students. If we let x be the number of girls at Wintrhop Middle School and y the total number of students this means that

$$y = \frac{9}{5}x.$$

We are given that there are 180 students at Colfax Middle School. Setting $y = 180$ we can solve for x to find that there are $180 \times \frac{5}{9} = 100$ girls at Colfax Middle School.

At the two schools combined there are $120 + 100 = 220$ girls. So the fraction of students at the dance who are girls is
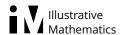
$$\frac{220}{220 + 230} = \frac{220}{450} = \frac{22}{45}.$$

Edit this solution

Solution: 3 Ratio Table

We can set up a ratio table to find the number of girls in each school. Here is an example of Colfax Middle School:

4	9
40	90

80	180
120	270

Students may fill out more or less information in the table. The key is to get 270 in the second column since there are 270 students at Colfax Middle School.

Next we set up a ratio table to find the number of girls at Wintrhop Middle School:

5	9
50	90
100	180

This shows that 100 of the students at Winthrop Middle School are girls.

At the two schools combined there are $120 + 100 = 220$ girls. So the fraction of students at the dance who are girls is

$$\frac{220}{220+230} = \frac{220}{450} = \frac{22}{45}.$$