8.EE Sore Throats, Variation 2

Alignments to Content Standards: 8.EE.B.5

Task

Nia and Trey both had a sore throat so their mom told them to gargle with warm salt water.

> Nia mixed 1 teaspoon salt with 3 cups water.
> Trey mixed $\frac{1}{2}$ teaspoon salt with $1\frac{1}{2}$ cups of water.

Nia tasted Trey's salt water. She said,

"I added more salt so I expected that mine would be more salty, but they taste the same."

a. Explain why the salt water mixtures taste the same.

b. Find an equation that relates s, the number of teaspoons of salt, with w, the number of cups of water, for both of these mixtures.

c. Draw the graph of your equation from part b.

d. Your graph in part c should be a line. Interpret the slope as a unit rate.

IM Commentary

The purpose of this task is to show how the ideas in the RP and EE domains in 6th and

7th grade mature in 8th grade. Parts (a)-(c) could easily be asked of 7th grade students. Part (a) asks students to do what is described in 7.RP.2.a, Part (b) asks students to do what is described in 7.RP.2.c, and Part (c) is the 7th grade extension of the work students do in 6.EE.9.

On the other hand, part (d) is 8th grade work. It is true that in 7th grade, "Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope" (CCSSM p. 46). However, in 8th grade students are ready to treat slopes more formally: 8.EE.5 says students should "graph proportional relationships, interpreting the unit rate as the slope of the graph" which is what they are asked to do in part (d).

There is a non-mathematical fact that students must know about mixtures in order to answer this question. When salt is dissolved in water, the salt disperses evenly through the mixture, so any sample from the mixture that has the same volume will have the same amount of salt. This is not something that kids could know a priori or by reasoning about it. For example, the same is not true when you mix sand and water. In general, it is important to know what facts about the world warrant applying a particular mathematical structure in a given context. In this particular case, teachers may need to provide some background knowledge or help students explain why a ratio is an appropriate mathematical tool in this context.

Solutions

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Solution: Finding equivalent ratios

a. The ratio of the number of teaspoons of salt to the number of cups of water is 1:3 in Nia's solution. If we divide the amount of salt and the amount of water by 3, the ratio will be the same.

$$1 \div 3 = \frac{1}{3}$$

$$3 \div 3 = 1$$

Thus 1:3 is equivalent to the ratio $\frac{1}{3} : 1$, which means that Nia's solution has $\frac{1}{3}$ teaspoon of salt for every cup of water. The ratio of the number of teaspoons of salt to the number of cups of water is $\frac{1}{2} : 1\frac{1}{2}$ in Trey's solution. If we divide the amount of salt

and the amount of water by $1\frac{1}{2}$, the ratio will be the same.

$$\frac{1}{2} \div 1\frac{1}{2} = \frac{1}{3}$$

$$1\frac{1}{2} \div 1\frac{1}{2} = 1$$

So Trey's ratio is also equivalent to the ratio $\frac{1}{3} : 1$. Since each mixture has the same amount of salt for every 1 cup of water, they are equally salty.

b. One equation is $s = \frac{1}{3}w$. An equivalent equation is $w = 3s$.

c. If the number of cups of water is represented on the horizontal axis, then the graph will be a line through (0,0) and (3,1). If the number of teaspoons of salt is represented on the horizontal axis, then the graph will be a line through (0,0) and (1,3).

d. If the number of cups of water is represented on the horizontal axis, then the slope is 1/3 which means that there is 1/3 teaspoon of salt per cup of water. If the number of teaspoons of salt is represented on the horizontal axis, then the slope is 3 which means that there are 3 cups of water per teaspoon of salt.

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Solution: Another way to solve the first part

a. Another, simpler way to solve the first part is to note that if you divide both quantities in Nia's ratio by 2, you get Trey's ratio. You will still have to interpret the unit rate for the other parts of the task.

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